Transverse nonlinear dynamics of axially accelerating viscoelastic beams based on 4-term Galerkin truncation

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Accepted 1 April 2005

Abstract

This paper investigates bifurcation and chaos in transverse motion of axially accelerating viscoelastic beams. The Kelvin model is used to describe the viscoelastic property of the beam material, and the Lagrangian strain is used to account for geometric nonlinearity due to small but finite stretching of the beam. The transverse motion is governed by a nonlinear partial-differential equation. The Galerkin method is applied to truncate the partial-differential equation into a set of ordinary differential equations. When the Galerkin truncation is based on the eigenfunctions of a linear non-translating beam subjected to the same boundary constraints, a computation technique is proposed by regrouping nonlinear terms. The scheme can be easily implemented in practical computations. When the transport speed is assumed to be a constant mean speed with small harmonic variations, the Poincaré map is numerically calculated based on 4-term Galerkin truncation to identify dynamical behaviors. The bifurcation diagrams are present for varying one of the following parameter: the axial speed fluctuation amplitude, the mean axial speed and the beam viscosity coefficient, while other parameters are unchanged.

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1. Introduction

Many engineering devices, such as serpentine belts and band saws, involve transverse motion of axially moving beams. The traditional investigations on axially moving beams were focused on equilibriums and periodic motions [1,2]. Modern nonlinear dynamics reveals that bifurcation means the sudden changes of dynamical behaviors with the system parameters and chaos implies motions with the continuous frequency spectrum. Both of them may be significant in applications.

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Recently, some attention has been paid to nonlinear dynamical behaviors, including bifurcation and chaos, in axially moving beams. Using the 1-term Galerkin truncation, Ravindra and Zhu studied pitchfork bifurcation and chaos of axially accelerating beams in the supercritical regime [3]. Pellicano and Vestroni carried out bifurcation and stability analysis for an axially moving beam based on 4-term Galerkin truncation [4]. Öz et al. applied the direct method of multiple scales to vibration of an axially accelerating beam, and analyzed stability and bifurcation of steady-state solutions [5]. Pellicano and Vestroni numerically studied bifurcation and chaos in an axially moving beam with transverse load, and found that a few number of degree-of-freedom is sufficient to furnish a good spatial representation and to follow actual dynamical behaviors [6]. In the aforementioned research on nonlinear problems of axially moving beams, the material of the beams were assumed to be elastic, and the damping effect was neglected or modeled as a simple viscous damping mechanism. The modeling of dissipative mechanisms is an important research topic of axially moving material vibrations [1,2]. Viscoelasticity is an effective approach to model the dissipative mechanism. Besides, their modeling was based on the quasi-static stretch assumption. Chen and Yang demonstrated that that model is only an approximate one for transverse motion [7].

As documented by previous studies, the Galerkin truncation is an important numerical approach to investigate nonlinear dynamics of axially moving viscoelastic beams. So far, all numerical investigations on bifurcation and chaos of axially moving beams [3,6,8–11] used the eigenfunctions of a linear non-translating beam as the base of the Galerkin truncation. In the application of the Galerkin truncation, the main problem in the actual computations is the complicity in the resulting truncated equations, especially those without the quasi-static stretch assumption. If the number of terms retained in the Galerkin truncation is rather large, the explicit expression of nonlinear terms is very difficult to obtain. To address the problem, this paper proposes a technique to simplify the nonlinear terms in the equations derived from the Galerkin truncation. All nonlinear terms are regrouped to combine the repeated terms and cancel the zero terms. Therefore, the resulting equations are easily coded for computers and then effectively calculated. The proposed technique is applied to study bifurcation and chaos in transverse motion of axially accelerating viscoelastic beams based on 4-term Galerkin truncation.

2. The governing equation

A uniform axially moving viscoelastic beam, with density $\rho$, cross-sectional area $A$, moment of inertia $I$ and initial tension $P_0$, travels at the time-dependent axial transport speed $v(T)$ between two prismatic ends separated by distance $L$. Consider only the bending vibration described by the transverse displacement $U(X, T)$, where $T$ is the time and $X$ is the axial coordinate. The viscoelastic material of the beam obeys the Kelvin model,

$$\sigma(X, T) = E\varepsilon_k(X, T) + \eta \frac{\partial \varepsilon_k(X, T)}{\partial T}$$

(1)

where $\sigma(X, T)$ is the axial disturbed stress and $\varepsilon_k(X, T)$ is the Lagrangian strain to account for geometric nonlinearity due to small but finite stretching of the beam.

Chen and Yang derived the governing equation of transverse motion from the Newton's second law and the linear moment–curvature relationship for slender beams [7]

$$\rho A \left( \frac{\partial^2 U}{\partial T^2} + 2v \frac{\partial U}{\partial T} + \frac{dv}{dT} \frac{\partial U}{\partial X} + v^2 \frac{\partial^2 U}{\partial X^2} \right) - P_0 \frac{\partial^2 U}{\partial X^2} + \frac{EI}{L} \frac{\partial^4 U}{\partial X^4} + \eta \frac{\partial^3 U}{\partial T^2} \frac{\partial U}{\partial X} \\
= \frac{3E}{A} \left( \frac{\partial U}{\partial X} \right)^2 \frac{\partial^2 U}{\partial X^2} + 2\eta \left( \frac{\partial U}{\partial X} \right) \frac{\partial^2 U}{\partial X \partial T} + \eta \left( \frac{\partial U}{\partial X} \right)^2 \frac{\partial^3 U}{\partial X^2 \partial T} \tag{2}$$

Introduce the dimensionless variables and parameters
\[ u = \frac{U}{L}, \quad x = \frac{X}{L}, \quad t = T \sqrt{\frac{P_0}{\rho A L^2}}, \quad \gamma = v \sqrt{\frac{P_0}{\rho A}}, \quad k_i^2 = \frac{EI}{P_0 L^2} \]

\[ \alpha = \frac{I \eta}{L^3 \sqrt{\rho A P_0}}, \quad k_1 = \frac{EA}{P_0}, \quad k_2 = \frac{A \eta}{L \sqrt{P_0 \rho A}} \]

Eq. (2) can be cast into the dimensionless form

\[ \frac{\partial^2 u}{\partial x^2} + 2\gamma \frac{\partial^2 u}{\partial x \partial t} + \frac{d^2 u}{dt^2} + (\gamma^2 - 1) \frac{\partial^2 u}{\partial x^2} + k_1^2 \frac{\partial^4 u}{\partial x^4} + \alpha \frac{\partial^2 u}{\partial x^2} \left( \frac{\partial u}{\partial x} \right)^2 + \varkappa k_2 \left[ \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 u}{\partial x^2} + \left( \frac{\partial u}{\partial x} \right)^2 \frac{\partial^4 u}{\partial x^4} \right] \]

In the present investigation, the beam is with the simple support at the both ends. The boundary conditions in the dimensionless form are

\[ u(0, t) = u(1, t) = 0, \quad \left. \frac{\partial^2 u}{\partial x^2} \right|_{x=0} = \left. \frac{\partial^2 u}{\partial x^2} \right|_{x=1} = 0 \]

3. Galerkin’s truncation

The Galerkin method will be used to solve numerically nonlinear partial-differential equation (4) under the boundary condition (5). Suppose that the solution to Eq. (4) takes the form

\[ u(x, t) = \sum_{i=1}^{\infty} q_i(t) \phi_i(x) \]

where \( q_i(t) \) is a set of generalized displacements of the beam and \( \phi_i(x) \) is a set of the trial functions satisfying the boundary conditions (5). Denote \( \psi(x) \) as a set of the weight functions. Then Eq. (4) is discretized into a set of infinite ordinary differential equations

\[ \left\{ \sum_{i=1}^{\infty} \hat{q}_i(t) \phi_i(x) + 2\gamma \sum_{i=1}^{\infty} \hat{q}_i(t) \phi_i'(x) + \gamma^2 - 1 \sum_{i=1}^{\infty} q_i(t) \phi_i''(x) + k_1^2 \sum_{i=1}^{\infty} q_i(t) \phi_i^{(4)}(x) \right\} \]

\[ + \alpha \sum_{i=1}^{\infty} \hat{q}_i(t) \phi_i^{(4)}(x) - \frac{3}{2} k_1^2 \sum_{i=1}^{\infty} q_i(t) \phi_i''(x) \left[ \sum_{i=1}^{\infty} q_i(t) \phi_i''(x) \right]^2 - \varkappa k_2 \left\{ 2 \sum_{i=1}^{\infty} q_i(t) \phi_i'(x) \sum_{i=1}^{\infty} \hat{q}_i(t) \phi_i'(x) \sum_{i=1}^{\infty} q_i(t) \phi_i'(x) \right\} \]

\[ + \left[ \sum_{i=1}^{\infty} q_i(t) \phi_i'(x) \right] \left[ \sum_{i=1}^{\infty} \hat{q}_i(t) \phi_i'(x) \right] \psi_n(x) = 0 \quad (n = 1, 2, \ldots) \]

where the inner production for two functions \( f \) and \( g \) on \([0, 1]\), is defined as

\[ \langle f, g \rangle = \int_0^1 f(x) g(x) dx \]

Application of the distributive law of the inner product to Eq. (7) leads to

\[ \sum_{i=1}^{\infty} \hat{q}_i(t) \langle \phi_i, \psi_n \rangle + 2\gamma \sum_{i=1}^{\infty} \hat{q}_i(t) \langle \phi_i', \psi_n \rangle + \gamma^2 - 1 \sum_{i=1}^{\infty} q_i(t) \langle \phi_i'', \psi_n \rangle + k_1^2 \sum_{i=1}^{\infty} q_i(t) \langle \phi_i^{(4)}, \psi_n \rangle + \alpha \sum_{i=1}^{\infty} \hat{q}_i(t) \langle \phi_i^{(4)}, \psi_n \rangle \]

\[ - \frac{3}{2} k_1^2 \sum_{i=1}^{\infty} q_i(t) \langle q_i(t) \rangle \langle \phi_i'', \phi_i'' \rangle - \varkappa k_2 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} q_i(t) \langle q_i(t) \rangle \langle q_j(t) \phi_j \phi_k \rangle \psi_n = 0 \quad (n = 1, 2, \ldots) \]
Truncating Eq. (9) and retaining first \( m \) terms yield

\[
\sum_{i=1}^{m} \ddot{q}_i(t)\langle \phi_i, \psi_n \rangle + 2\gamma \sum_{i=1}^{m} \dot{q}_i(t)\langle \phi_i', \psi_n \rangle + (\gamma^2 - 1) \sum_{i=1}^{m} q_i(t)\langle \phi_i^q, \psi_n \rangle + k_2 \sum_{i=1}^{m} q_i(t)\langle \phi_i^q, \psi_n \rangle + \alpha \sum_{i=1}^{m} \ddot{q}_i(t)\langle \phi_i^m, \psi_n \rangle
\]

\[
- \frac{3}{2} k_2 \sum_{i=1}^{m} \sum_{j=1}^{m} q_i(t)q_j(t)\langle \phi_i'\phi_j', \psi_n \rangle - 2\alpha k_2 \sum_{i=1}^{m} \sum_{j=1}^{m} q_i(t)q_j(t)\dot{q}_i(t)
\]

\[
\times \left( \langle 2\phi_i'\phi_j'\phi_k', \psi_n \rangle + \langle \phi_i'\phi_j'\phi_k', \psi_n \rangle \right) = 0 \quad (n = 1, 2, \ldots, m)
\]

which is set of \( m \) ordinary differential equations.

In the present investigation, like in the previous studies [3,6,8–11], both the trial and weight functions are chosen as the eigenfunctions of a linear non-translating beam under the boundary condition (5), namely,

\[
\phi_i(x) = \sin(i\pi x), \quad \psi_n(x) = \sin(n\pi x) \quad (i, n = 1, 2, \ldots, m)
\]

Insetting Eq. (11) into Eq. (10), evaluating the corresponding inner products, regrouping the nonlinear terms to combine the same terms and canceling all null terms in the resulting equation, one obtains

\[
\ddot{q}_n(t) + 4 \sum_{i=1}^{m} \frac{m_i}{n_i + 1} [2\gamma \dot{q}_i(t) + \dot{q}_i(t)] + (\gamma^2 - 1)n^2\pi^2 q_n(t) + k_2 n^2\pi^4 q_n + 2n^3\pi^4 \dot{q}_n = \frac{7n\pi^4}{16}
\]

\[
\times \left\{ \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} \left[ i(1-i)q_i(k_i q_j + 2k_2 q_j) \right] \right\} + \sum_{i=1}^{m} \left\{ (n+1)q_{n+1} (1-i)q_i(k_i q_j + 2k_2 q_j) \right\}
\]

\[
- \frac{5n^3\pi^4}{16} \sum_{i=1}^{m} \left( (n+1)q_{n+1} (1-i)q_i(k_i q_j + 2k_2 q_j) \right) - \frac{nk_2 n^3\pi^4}{8} \sum_{i=1}^{m} \left( (n+1)q_{n+1} (1-i)q_i(k_i q_j + 2k_2 q_j) \right)
\]

\[
+ \sum_{i=1}^{m} \left( (n+1)q_{n+1} (1-i)q_i(k_i q_j + 2k_2 q_j) \right) + \sum_{i=1}^{m} \left( (n+1)q_{n+1} (1-i)q_i(k_i q_j + 2k_2 q_j) \right)
\]

where the sum is defined to be zero if its lower limit is larger than its upper limit. Although Eq. (12) seems rather complicated; it is very efficient when used for computer implementing, because almost all repeated nonlinear terms are put together, and terms with zero coefficients are eliminated. In fact, Eq. (10) contains \( 2m^3 \) nonlinear terms, while Eq. (12) contains less than \( 2m^2 \) nonlinear terms. For large, the difference is significant.

For specified initial condition

\[
u(x, 0) = a(x), \quad \frac{\partial u}{\partial t}(x, 0) = b(x)
\]

Fig. 1. Bifurcation versus the dimensionless speed fluctuation amplitude: (a) displacement and (b) velocity.
the initial generalized displacements and velocities for Eq. (12) are respectively
\[
q_n(0) = 2 \int_0^1 a(x) \sin(n\pi x) dx, \quad \dot{q}_n(0) = 2 \int_0^1 b(x) \sin(n\pi x) dx \quad (n = 1, 2, \ldots, m)
\]
Thus Eq. (12) with initial condition (14) is an $m$ term Galerkin truncation of Eq. (4) with boundary condition (5) and initial condition (14).

Fig. 5. Local magnification of Fig. 4: period-doubling bifurcation. (a) Displacement and (b) velocity.

Fig. 6. Bifurcation versus the dimensionless viscosity coefficient: (a) displacement and (b) velocity.

Fig. 7. Local magnification of Fig. 6: backward period doubling bifurcation. (a) Displacement and (b) velocity.
In the numerical calculations, choose $m = 4$. Eq. (12) can be written in an explicit form without sum notations

\[
\begin{align*}
\ddot{q}_1 - \pi^2 (\gamma^2 - 1)q_1 + v^2 \pi^4 q_1 + z x^4 q_1 &= \frac{8}{3}\gamma q_2 - \frac{16}{15}\gamma q_4 - \frac{16}{3}\gamma q_2 - \frac{32}{15}\gamma q_4 \\
+ k_1^2 \pi^4 \left( \frac{8}{3}q_1 + 3q_1 q_2^2 + \frac{9}{2}q_1 q_3 + \frac{27}{4}q_1 q_3^2 + 6q_1 q_3 q_4 + 18q_1 q_3 q_4 + 12q_1 q_4^2 \right) \\
+ k_2^2 \pi^4 \left[ \left( \frac{3}{4}q_1^2 + 2q_2^2 + \frac{3}{4}q_1 q_3 + \frac{9}{2}q_3^2 + 4q_2 q_3 + 8q_2^2 \right)q_1 + (4q_1 q_2 + 6q_1 q_3 + 4q_1 q_4 + 12q_1 q_4)q_1 \\
+ \left( \frac{3}{4}q_1^2 + 3q_2^2 + 9q_1 q_3 + 12q_1 q_4 \right)q_1 + (4q_1 q_2 + 12q_1 q_3 + 16q_1 q_4)q_1 \right] &= 0 \\
\ddot{q}_2 - \pi^2 (\gamma^2 - 1)q_2 + 16v^2 \pi^4 q_2 + 16x^4 q_2 + \frac{8}{3}\gamma q_3 - \frac{24}{5}\gamma q_3 + \frac{16}{3}\gamma q_1 + \frac{48}{5}\gamma q_4 \\
+ k_1^2 \pi^4 \left( 3q_1 q_2^2 + 6q_1^3 + 9q_1 q_3 + 3q_1 q_3^2 + 27q_2 q_3^2 + 18q_1 q_3 q_4 + 27q_1 q_3 q_4 + 48q_1 q_3^2 \right) \\
+ k_2^2 \pi^4 \left[ (4q_1 q_2 + 6q_1 q_3 + 4q_1 q_4 + 12q_1 q_4)q_1 + (2q_1^2 + 12q_2^2 + 6q_1 q_3 + 18q_1^2 + 32q_1^2)q_2 \\
+ (6q_1 q_2 + 36q_2 q_3 + 12q_1 q_4 + 36q_4 q_3 + 12q_1 q_4 + 18q_1 q_4 + 64q_4 q_4)q_1 \right] &= 0 \\
\ddot{q}_3 - 9\pi^2 (\gamma^2 - 1)q_3 + 81v^2 \pi^4 q_3 + 81x^4 q_3 + \frac{24}{5}\gamma q_3 - \frac{48}{7}\gamma q_3 - \frac{48}{5}\gamma q_2 - \frac{96}{7}\gamma q_4 \\
+ k_1^2 \pi^4 \left( \frac{3}{8}q_1^2 + \frac{9}{2}q_1 q_2^2 + \frac{27}{4}q_1 q_3^2 + \frac{243}{8}q_1 q_3^2 + 18q_1 q_2 q_4 + 54q_1 q_3 q_4 + 108q_1 q_3 q_4 \right) \\
+ k_2^2 \pi^4 \left[ \left( \frac{3}{4}q_1^2 + 3q_2^2 + 9q_1 q_3 + 12q_1 q_4 \right)q_1 + (6q_1 q_2 + 36q_2 q_3 + 12q_1 q_4 + 36q_4 q_3)q_2 \\
+ \left( \frac{9}{2}q_1^2 + 18q_2^2 + \frac{243}{4}q_3^2 + 36q_2 q_3 + 72q_4^2 \right)q_3 + (12q_1 q_2 + 36q_2 q_3 + 144q_4 q_4)q_4 \right] &= 0 \\
\ddot{q}_4 - 16\pi^2 (\gamma^2 - 1)q_4 + 256\pi^4 q_4 + 256x^4 q_4 + \frac{16}{15}\gamma q_1 + \frac{48}{7}\gamma q_3 + \frac{32}{15}\gamma q_1 + \frac{96}{7}\gamma q_3 \\
+ k_1^2 \pi^4 \left( 3q_1 q_2^2 + 18q_1 q_3 + 27q_2 q_3 + 12q_2 q_4 + 48q_2 q_4 + 108q_1 q_4 + 96q_1 q_4 \right) \\
+ k_2^2 \pi^4 \left[ (4q_1 q_2 + 12q_2 q_3 + 16q_1 q_4)q_1 + (2q_1^2 + 12q_2^2 + 18q_1 q_3 + 64q_4 q_4)q_2 \\
+ (12q_1 q_2 + 36q_2 q_3 + 144q_4 q_4)q_3 + (8q_1^2 + 32q_1^2 + 72q_3^2 + 192q_4)q_4 \right] &= 0
\end{align*}
\]

\section{Bifurcation and chaos}

In the following, the dimensionless axial speed is assumed to be a small simple harmonic variation, with the amplitude $\gamma_1$ and the frequency $\omega$, about the constant mean speed $\gamma_0$.

\[
\gamma(t) = \gamma_0 + \gamma_1 \sin \omega t
\]

Under the assumption, Eq. (15) describes a nonlinear parametrically excited system.

The Poincaré map will be used to identify the dynamical behavior. The bifurcation diagrams of the Poincaré map are calculated to view globally over a range of parameter values and compare simultaneously regular and chaotic motions. The Poincaré map is constructed by computing the dimensionless displacement and velocity of the center of the moving beam, which can be determined based on numerical integration of Eq. (15). In fact, the dimensionless displacement and velocity of the center derived from Eqs. (6) and (11) are respectively

\[
\begin{align*}
\ddot{u} \left( \frac{1}{2}, nT \right) &= q_1 (nT) - q_3 (nT), \quad \dot{u} \left( \frac{1}{2}, nT \right) = q_1 (nT) - q_3 (nT)
\end{align*}
\]
tude of axial speed fluctuation is larger than a certain value. With the increasing amplitude, after a series of period-doubling bifurcation, chaotic motion appears. However, with the further increase of the amplitude, chaotic motion disappears suddenly, and periodic motion occurs again. Finally, when the amplitude is sufficiently large, the periodic motion bifurcates into chaos. Fig. 2, a local magnification of Fig. 1, depicts a typical period-doubling bifurcation sequence. Fig. 3, another local magnification of Fig. 1, details different period windows in the chaotic range. Therefore, the chaotic motion and the periodic motion exchange alternately in both large and small scales.

Fig. 4 is the bifurcation diagrams via the meal axial speed $c_0$, in which $k_f = 0.8$, $k_1 = 0.8$, $k_2 = 1.0$, $c_1 = 1.0$, and $x = 3.5$. For the small mean speed, there is a stable equilibrium. With the increase of the mean speed, the equilibrium may become unstable and alternates with a periodic motion. With the further increase of the mean speed, period-doubling bifurcation occurs, which is shown in the locally magnified diagrams, Fig. 5. For the mean speed large enough, chaotic motion appears.

Fig. 6 is the bifurcation diagrams via the viscosity coefficient $x$, in which $k_f = 0.8$, $k_1 = 0.8$, $k_2 = 1.0$, $c_0 = 1.0$, $c_1 = 1.0$, and $x = 3.5$. Chaotic motion occurs for the small viscosity coefficient. Within the chaotic range, there are some period windows. When the viscosity coefficient is adequately large, chaotic motion is replaced by periodic motion through backward period doubling, shown in Fig. 7. With the further increase of the viscosity coefficient, a stable equilibrium appears.

Fig. 8 illustrates a period doubling bifurcation represented by phase trajectories, in which $k_f = 0.8$, $x = 0.001$, $k_1 = 0.8$, $k_2 = 1.0$, $c_0 = 2.4$, and $x = 3.5$. For $c_1 = 0.5$, 0.68, 0.692, there are respectively period-1, period-2 and period-4 motions. A typical chaotic motion identified by the phase trajectory, the Poincaré map, and the largest Lyapunov exponent is demonstrated in Fig. 9 in which $k_f = 0.8$, $x = 0.001$, $k_1 = 0.8$, $k_2 = 1.0$, $c_0 = 2.4$, $c_1 = 0.5$, and $x = 3.5$. 

![Fig. 8. Period-doubling bifurcation shown by phase trajectories: (a) period-1 motion, (b) period-2 motion and (c) period-4 motion.](image-url)
5. Conclusions

Bifurcation and chaos in transverse motion of an axially accelerating viscoelastic beams are investigated in this paper. The transverse motion of the beam is governed by a nonlinear partial-differential equation. The Galerkin’s method is applied to truncate the governing equation into a set of ordinary differential equations. When the eigenfunctions of a linear non-translating beam, which satisfy the boundary conditions, serve as the trial and weight functions, a technique is proposed to cast the truncated equations into a simple form by merging the repeated nonlinear terms and deleting the zero terms. The resulting equation is easy for computers to code. Based on the numerical solutions of the 4-term Galerkin truncation, the Poincaré map is constructed to classify the motions. The bifurcation diagrams are calculated in the case that the axial speed perturbation amplitude, the mean axial speed, or the viscosity coefficient is respectively varied while other parameters are fixed. Numerical results show that, with the increasing speed perturbation amplitude, the increasing mean speed, and the decreasing viscosity coefficient, the equilibrium loses its stability and bifurcates into a periodic motion, and the periodic motion becomes chaotic motion via period doubling bifurcation. In addition, the chaotic motion and the periodic motion exchange alternately for the sufficiently large speed perturbation amplitude and mean speed, and for the sufficiently small viscosity coefficient.

Acknowledgements

The research is supported by the National Natural Science Foundation of China (Project No. 10472060) and Natural Science Foundation of Shanghai Municipality (Project No. 04ZR14058).
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